



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 38

American Options – 1

J P Singh

Department of Management Studies



AMERICAN OPTIONS



EARLY EXERCISE: AMERICAN CALLS

- Suppose you hold an American call on a stock and decide to exercise it early at $t=\tau$. **You will do this only if $S_\tau > K$.** Then, you will get the underlying stock immediately at $t=\tau$ for a predetermined price K . There can be two scenarios:
 - (A) you decide to hold the underlying stock yourself till option maturity date $t=T$ or
 - (B) you decide to sell the underlying stock before maturity date of the option ($t=T$).



CASE (A): HOLDING THE UNDERLYING STOCK TILL MATURITY OF OPTION

- In this case, on the maturity date, your holdings shall consist of one unit of underlying stock and zero options since you have already exercised the option.
- You can achieve the same holdings on the maturity date by exercising the option on the maturity date itself.
- However, this strategy has two clear advantages viz.



- (i) Since, you pay the same predetermined price on the maturity date $t=T$, and not at the earlier date $t=\tau$, you save the interest cost for the period between early exercise $t=\tau$ and maturity date $t=T$ on the predetermined price K .
- (ii) You continue to enjoy insurance against a fall in the stock price below the exercise price i.e. if the stock price happens to fall below the exercise price at $t=T$, you still have the discretion not to exercise the option and restrict your loss to the option premium.
- But, if you have already exercised the option, then all the loss on account of the fall in stock price will be borne by you since you are now the owner of the stock. Option premium is also to be borne by you since you have bought the option.



CASE (B): SELLING THE UNDERLYING STOCK BEFORE MATURITY OF OPTION

- In this case, compare the two scenarios:
- (i) selling the stock after exercising the option
- (ii) selling the option without exercising
- In the latter strategy, the buyer can do everything embedded in the former strategy by simply exercising the option.
- However, he enjoys additional benefit of insurance against a price fall of the underlying if he wants to hold the option till maturity.
- The buyer will, therefore, prefer and pay more for (ii) than for (i).



- The option will be bought by another investor who does want to hold the stock. Such investors must exist, otherwise the current stock price would not be sustained. The price obtained for the option will, thus, be greater than its intrinsic value of $S_T - K$.



A QUANTITATIVE ARGUMENT

- We have shown that $\underline{c} \geq S_\tau - Ke^{-r(T-\tau)}$
- Also $C \geq \underline{c}$ so that $C \geq S_\tau - Ke^{-r(T-\tau)}$
- Since, $r > 0$, this implies $C > S_\tau - K$
- Thus, selling the American call without exercising will yield a higher profit than first exercising it and then selling the stock.
- If it were optimal to early exercise an American call, we would have $C = S_\tau - K$. Thus, it is never optimal to exercise early.



EARLY EXERCISE: AMERICAN PUTS

- Consider an American put option with exercise price K and maturity T . We need to establish the existence of at least one scenario such that it would be optimal to exercise the American put before time T .
- To understand this, recall that a put option provides protection against a price fall i.e. if the price of the asset falls below the exercise price, the put option holder can still sell the asset at the exercise price.



- Hence, consider a situation, say at time $t=\tau$ where $0<\tau<T$ at which the price of the underlying is almost zero. In this situation, why would you hold on to the American put? Only if you expect the price of the underlying to go down further in the time (τ,T) . But since the price is already close to zero, the chances of further fall are negligible.
- On the other hand, if you exercise the option at $t=\tau$, you can invest the receipts elsewhere and earn interest income.
- Hence, in such a situation it would be optimal to exercise the put option early.



AMERICAN OPTIONS: PUT CALL PARITY

	$t=0$	$t=\tau$	$t=T$	
PORTFOLIO A		$S_\tau < K$	$S_T < K$	$S_T > K$
BUY EUROPEAN CALL	$-c$	≥ 0	0	$S_T - K$
INVEST	$-K$	$Ke^{(r\tau)}$	$Ke^{(rT)}$	$Ke^{(rT)}$
TOTAL	$-(c+K)$	$\geq Ke^{(r\tau)}$	$Ke^{(rT)}$	$S_T + Ke^{(rT)} - K$

	$t=0$	$t=\tau$	$t=T$	
PORTFOLIO A		$S_\tau < K$	$S_T < K$	$S_T > K$
TOTAL	$-(c+K)$	$\geq Ke^{(r\tau)}$	$Ke^{(rT)}$	$S_T + Ke^{(rT)} - K$
PORTFOLIO B				
BUY STOCK	$-S_0$	S_τ	S_T	S_T
BUY AMERICAN PUT	$-P$	$K - S_\tau$	$K - S_T$	0
TOTAL	$-(S_0 + P)$	K	K	S_T
$S_0 + P \leq c + K$ but $c = C$, so $S_0 - K \leq C - P$				

From previous slide: $S_0 - K < C - P$

$$c + Ke^{(-rT)} = p + S_0$$

$$c - p = S_0 - Ke^{(-rT)}$$

$$c = C; p \leq P \text{ and } c, p > 0$$

$$C - P \leq c - p$$

$$C - P \leq S_0 - Ke^{(-rT)}$$

$$S_0 - K \leq C - P \leq S_0 - Ke^{(-rT)}$$

WHAT ABOUT $S_t > K$??

